**Abstract** - A novel Q-factor definition and evaluation method are proposed for low-loss high-Q spiral inductors fabricated by using the wafer level chip size package (WLP) on silicon substrates, where the copper wiring technology with a polyimide isolation layer is used. A complex conjugate impedance matching condition is retained both at an input port and an output port of the inductor. The maximum available power gain ($G_{AMAX}$) is introduced to evaluate the energy loss in one cycle. This condition provides a unique insertion loss of passive devices.

**Novel Q-factor definition and evaluation for a spiral inductor**

$$Q_{in} = 2\pi \frac{\text{Peak magnetic energy} - \text{Peak electric energy}}{\text{Energy loss in one cycle}}$$

$$= \frac{2\pi}{T} \frac{2(\text{Time average magnetic energy} - \text{Time average electric energy})}{\text{Input energy} \times \{1 - G_{AMAX}\}}$$

$$Q_{in} = \frac{[2 \text{Im}(S_{11})] \left[ 1 - S_{21} \Gamma_L^2 + 2 \text{Im}(S_{21}) \right] \left[ S_{22} \Gamma_L^2 + 4 \text{Im}(S_{22}) \text{Re}\{1 - S_{21} \Gamma_L \Gamma_L^* \} \right]}{[1 - |S_{11}|^2 - |S_{21}|^2 + |\Gamma_L|^2 \Delta^2 + |S_{22}|^2 - 2 \text{Re}\{\Gamma_L (S_{22} - \Delta S_{11})\}]}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

$$\Gamma_L = \Gamma_{in}^* = \left( S_{22} + \frac{S_{12} S_{22} \Gamma_L}{1 - S_{12} \Gamma_L} \right)^*$$

$$\Gamma_L = \Gamma_{out}^* = \left( S_{11} + \frac{S_{21} S_{11} \Gamma_L}{1 - S_{21} \Gamma_L} \right)^*$$

The difference is reduced!!

**High-Q spiral inductor fabricated by using wafer level CSP technology**

High-Q spiral inductor fabricated on a Si substrate

**Conventional definitions**

$$Q_{Port1} = -\frac{\text{Im}(Y_{11})}{\text{Re}(Y_{11})}$$

$$Q_{Port2} = -\frac{\text{Im}(Y_{12})}{\text{Re}(Y_{12})}$$

**Measured Q-factors derived from novel and conventional methods**